

# Geometry-confined bifurcation at low flow rate in flow-focusing droplet generator

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This communication describes the stable dripping (period-1)-bifurcation (period-2)-dripping (period-1) transition phenomenon in flow-focusing droplet generator. Different from the traditional higher-order periodic and chaotic bubbling which usually happened in the condition of high Weber number ( $We$ ), this transition is acquired under low flow rate. Experimental observations demonstrate this transition strongly depends on the orifice length, which indicates that the combination of capillary pressure and inertia dominates this regime.

Microdroplet technology has been exploited in the last two decades to provide diverse applications, especially in the areas of chemistry and biology, for example, food processing (Maan et al. 2015), targeted drug delivery (Van Wamel et al. 2016), diagnostic testing (El-Ali et al. 2006), and particle synthesis (Baah and Floyd-Smith 2014). In all microdroplet generation techniques, flow focusing has been proven to be a very effective and broad-spectrum method for generating the droplets and bubbles (Anna et al. 2003; Garstecki et al. 2005b; Gulati et al. 2016; Tan et al. 2006; Woodward et al. 2007; Xu and Nakajima 2004). In principle, the flow-focusing method is based on a co-flow that contains inner and outer two-phase fluids, in which the inner phase is pinched off by the outer phase to release droplets through an orifice.

Owing to the extensive application of flow-focusing droplet generator, many groups have studied the breakup modes of droplet to guide the devices designation. As soon

as Anna firstly described this complex process (Anna et al. 2003), various experiments, theories, and computations have been conducted to study these behaviors by changing geometry dimension (Gupta et al. 2014), fluids properties (Nie et al. 2008), flow rates (Garstecki et al. 2005b), and other parameters. Usually, droplet breakup regimes can be divided into squeezing, dripping, thread formation, and jetting (Lee et al. 2009). Though the monodisperse droplets are obtained in the above regimes, some polydispersity situations also exist. The most common one is dripping with satellite, a very small droplet induced by unbalance capillary force follows behind a primary droplet. Another case is the nonlinear phenomena in flow-focusing device including period-doubling, period-halving bifurcation, intermittency, and chaos (Garstecki et al. 2005a). Related nonlinear dynamic studies have been completed including nozzle injects (Liu et al. 2004), dripping faucet (Ambravaneswaran et al. 2000), and flow-focusing bubbling (Garstecki et al. 2005a). It was observed that the monodisperse bubbles or droplets were replaced by repeating sequences of two, four, or more fluid segments with different sizes when the flow rate of the dispersed phase or continuous phase exceeded a critical value. The period- $n$  ( $n = 1, 2, 3, 4, \dots$ ) was used to express the regimes of drop breakup, where  $n$  presented the branch number of droplet or bubble size. However, bifurcation almost happened under high flow rates in these studies. With the increase of the flow rate of inner phase or outer phase, the regimes changed from period 1–2–4... in turn.

In this communication, a stable period 1–2–1 transition under low flow rates was described. The nonlinear dynamics of flow-focusing bubble generator has been proved to be dominated by inertia (Garstecki et al. 2005a). We cannot help wondering whether the bifurcation under low flow rate is based on the same principle since the complex dripping behavior. Understanding the dynamics of this phenomenon

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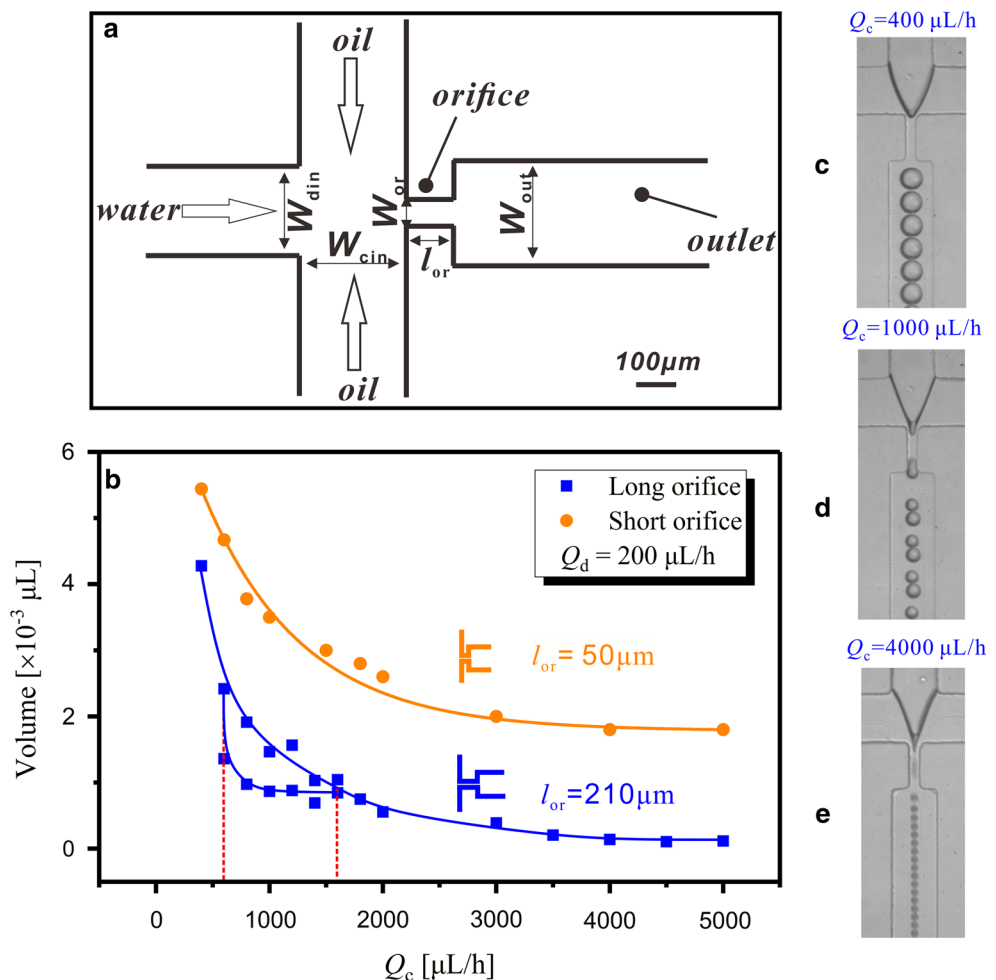
can help to control the range of parameters and gain uniform fluid segment.

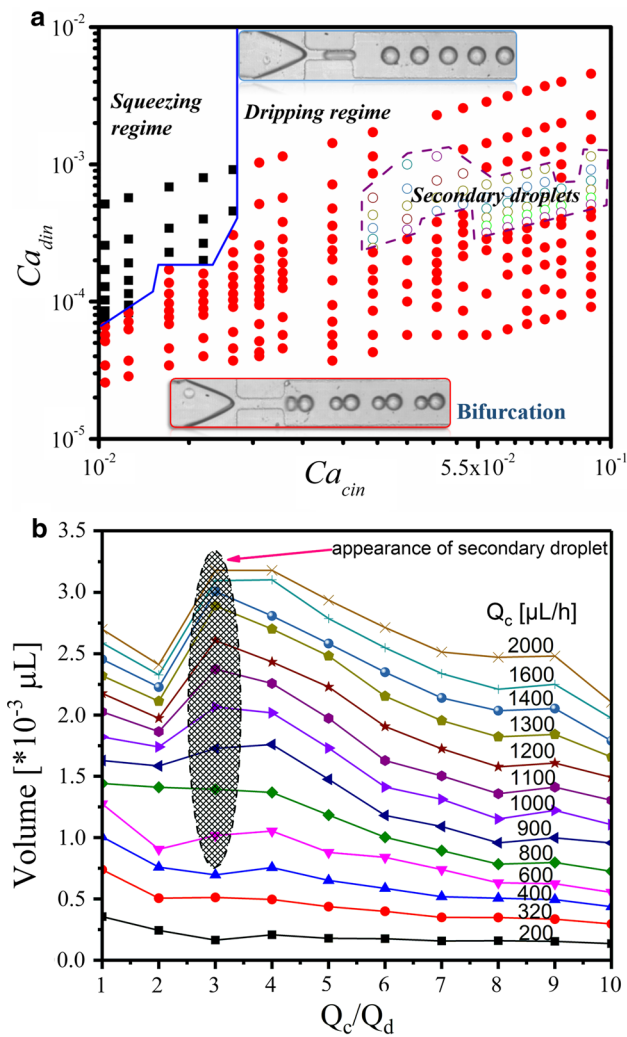
Figure 1a shows the flow-focusing devices used in the experiments. Devices with different orifice length ( $l_{or}$ ) were prepared by standard soft lithography techniques (Coulet et al. 2004) to investigate the origination of the bifurcation. These two channels have a uniform height of  $h = 72 \mu\text{m}$ , the width of the inlet of dispersed phase  $w_{din} = 200 \mu\text{m}$ , continuous phase  $w_{cin} = 217 \mu\text{m}$ , orifice  $w_{or} = 40 \mu\text{m}$ , and the length of the orifice  $l_{or} = 50 \mu\text{m}$  for short orifice flow-focusing device (SFF) and  $210 \mu\text{m}$  for long orifice flow-focusing device (LFF). Syringe pumps drove mineral oil as continuous phase and water as dispersed phase without any surfactant. An interface was formed between mineral oil and water at the cross-junction of the geometry. The oil flowed around the water during entering the orifice. Viscous and hydrostatic pressure together led the collapse of water thread. Interfacial tension kept the tip from moving forward. We marked the flow rate of mineral oil as  $Q_c$  and water as  $Q_d$ . At  $25^\circ\text{C}$ , the density and viscosity of deionized water and mineral oil are  $995 \text{ kg/m}^3$  and  $1 \text{ mPa s}$ ,  $817 \text{ kg/m}^3$  and  $40 \text{ mPa s}$ , respectively. The interfacial

tension of the two immiscible fluids is  $37.5 \text{ mN/m}$  measured by pendant drop method. Figure 1b illustrates the volume of droplets as a function of  $Q_c$  when  $Q_d$  ( $200 \mu\text{L/h}$ ) was a constant. All droplets in SFF were monodispersed as  $Q_c$  increased. While in LFF, monodispersed droplets (Fig. 1c) firstly turned to bifurcation (Fig. 1d, a primary droplet was followed by a relatively small secondary droplet), then the system produced identical droplets (Fig. 1e) again. In a particular  $Q_c$  range, the volume of droplets fluctuated with the increase of  $Q_c$  when  $Q_d$  fixed at  $200 \mu\text{L/h}$  (Fig. 1b), which denoted the bifurcation regime in LFF device.

To investigate the formation process of the dripping–bifurcation–dripping transition in LFF device, detailed experiments were designed by increasing  $Q_c$  from 160 to  $2000 \mu\text{L/h}$  and  $Q_d$  accordingly. For example,  $Q_d = 160, 80, \dots, 16 \mu\text{L/h}$  ( $Q_c = 160 \mu\text{L/h}$ ) that meant the flow rate ratio ( $Q_c/Q_d$ ) was increased from 1 to 10. Figure 2a shows the typical flow map which was based on the inlet capillary numbers of each fluids. Here the crucial superficial velocity of liquid and inlet capillary number are defined as  $u_j = Q_j/w_{jin}h$ ,  $Ca_{jin} = \eta_j u_j/\gamma$ , respectively. The subscript “j”

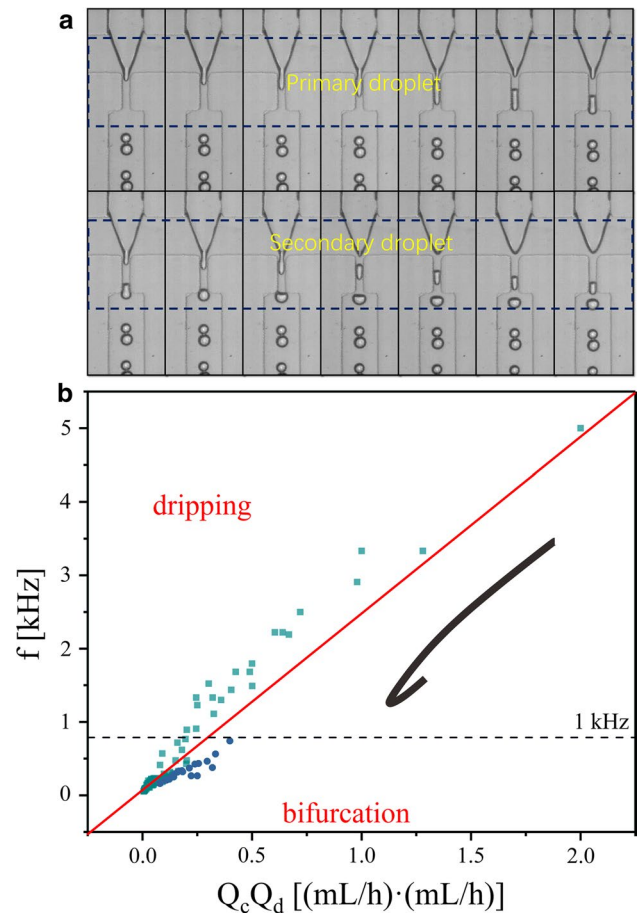
**Fig. 1** a The plane sketch map of the flow-focusing geometry. The channels have a uniform height of  $h = 72 \mu\text{m}$ , the width of the inlet of dispersed phase and continuous phase,  $w_{din} = 200 \mu\text{m}$  and  $w_{cin} = 217 \mu\text{m}$ , the width of the orifice,  $w_{or} = 40 \mu\text{m}$ , the length of the orifice,  $l_{or} = 50 \mu\text{m}$  for SFF and  $210 \mu\text{m}$  for LFF. b Droplets volume as a function of flow rate of continuous phase ( $Q_c$ ) under fixed flow rate of dispersed phase ( $Q_d = 200 \mu\text{L/h}$ ). For SFF geometry, all droplets generated in dripping area. For LFF geometry, dripping–bifurcation–dripping transition appeared. c, d, e The transition of LFF where  $Q_c$  increased from 400 to  $4000 \mu\text{L/h}$





**Fig. 2** **a** Flow map of LFF droplet generator about the inlet capillary number space of two phases (squeezing regime, dripping regime, and secondary droplets). The micrograph insets show the typical modes of the two regimes. Flow rate of continuous phase  $Q_c$  was taken from 200 to 2000  $\mu\text{L/h}$ , and flow rate ratio  $Q_c/Q_d$  was chosen from 1 to 10. **b** Droplets volume versus  $Q_c/Q_d$  under various  $Q_c$ . Primary and secondary droplets were taken as a whole to calculate the volume for bifurcation. The oval shadow area indicates the appearance of secondary droplet

in the formulas can be divided into two cases, “c” (continuous phase) and “d” (dispersed phase). The subscript “in” means “inlet.”  $\eta$  denotes viscosity of fluid, and  $\gamma$  is interfacial tension of two immiscible fluids. For example,  $Ca_{cin}$  means the inlet capillary number of continuous phase. The flow map showed a comprehensive dripping (period-1) regime with throughout  $Ca_{din} < 10^{-2}$ . When the value of  $Ca_{cin}$  was relatively low, the regime of the flow-focusing devices was squeezing or dripping (no secondary droplets) with  $Ca_{din}$  increasing. With the increase of  $Ca_{cin}$ , bifurcation (period-2) started to appear. At a particular  $Ca_{cin}$ , bifurcation only occurred within a certain  $Ca_{din}$  range.



**Fig. 3** **a** Breakup process of a pair of primary and secondary droplets. **b** Frequency of droplets plotted against the product ( $Q_c Q_d$ ). Outline of novel frequency bifurcation is shown as the solid line (not to scale). The short arm represented bifurcation regime, and the long arm represented dripping regime

Figure 2b shows the droplets volume versus flow rate ratio when  $Q_c$  was kept as a constant. Primary and secondary droplets were considered as a whole here to calculate droplet volume. In general, droplet volume first decreased with increase in  $Q_c/Q_d$ . Then, a sudden increase in volume was found, which indicated the presence of secondary droplets. The shadow area in Fig. 2b showed that all bifurcation appeared when  $Q_c/Q_d$  was about 3. Actually, the bifurcation still existed when the value of  $Q_c$  exceeded 3000  $\mu\text{L/h}$ . In conclusion, the flow map (Fig. 2a) and volume map (Fig. 2b) showed a minimal value of  $Ca_{cin}$  about  $10^{-2}$  for the possibility of bifurcation. During the appearance of bifurcation, the  $Q_c/Q_d$  was around 3. Under these prerequisites, increasing  $Q_c$  or  $Q_d$  led to bifurcation.

In order to study the origination dynamics of secondary droplets, the micrographs of generation process were firstly focused (Fig. 3a). In each period, tip of dispersed phase began to grow and form a thread near the entrance of orifice. Hydrodynamic pressure, capillary stresses, viscous

forces, and inertia forces worked together to induce the breakup of primary droplet in the orifice. Primary droplet started to move along the orifice toward downstream. Before the primary droplet left the orifice, the tip grew again and a smaller secondary droplet was induced. There was no visible contraction of the tip in the generation gap of primary and secondary droplets. Then, the thread retracted and the sequence repeated.

It is important to determine the physical parameters that control the transition from dripping to bifurcation. A flow-focusing device is always divided into three parts: inlet channel, orifice, and outlet channel. Obviously, the occurrence of the bifurcation should be attributed to the dynamics of the thread in the part of orifice and has nothing to do with the entrances and the outlet. The micrographs (Fig. 3a) of breakup process manifested that no perturbation was occurred during the formation of primary and secondary droplets. During the formation of the secondary droplet, the primary droplet was still within (or close to) the orifice. Therefore, the evolution of two immiscible phase interface in the orifice was crucial. The frequency  $f$  of droplet breakup was introduced to study the interface. There is also a bifurcation of frequency ( $f$ ) relative to the product of  $Q_c$  and  $Q_d$  (Fig. 3b). The product of  $Q_c$  and  $Q_d$  (like  $We$ 's " $u^2$ ") was used to consider the combined effect of flow rate of dispersed phase and continuous phase. The long arm represented dripping regime, and the short arm represented bifurcation regime. Frequency of the bifurcation regime was always below the critical value of  $f_{cr} = 1$  kHz, which indicated this bifurcation occurred at a relatively low frequency.

The dimensionless parameter capillary number ( $Ca$ ) and Weber number ( $We$ ) are used to characterize the relative effects of viscous forces and inertia forces on interfacial tension. To analyze the flow condition in the orifice,  $Ca_{flow}$  and  $We_{flow}$ , calculated on the basis of the mean speed of the liquid through the orifice, were introduced to compare the viscous and the inertial term exerted on the interface of growing thread and continuous phase of the flowing liquids with capillary pressure.  $Ca_{intfc}$  and  $We_{intfc}$ , calculated on the basis of the frequency of droplet generation and the typical length scale of the evolution of the water–oil interface, were introduced to distinguish the shear stresses and inertial term exerted on the interface of recoil tip and continuous phase counteracting the capillary pressure.

The mean speed of the liquid through the orifice is  $u_{flow} = Q/w_{or}h$  ( $Q = Q_c + Q_d$ ,  $w_{or} = 40$   $\mu\text{m}$ ,  $h = 72$   $\mu\text{m}$ , shown in Fig. 1a). Thus,  $Ca_{flow} = \mu u_{flow}/\gamma$  and  $We_{flow} = \rho u_{flow}^2 h/\gamma$  ( $\mu = 40$  mPa s,  $\rho = 817$  kg/m<sup>3</sup>,  $\gamma = 37.5$  mN/m). In our experimental flow rates range,  $Ca_{flow} \in (10^{-2}, 10^{-1})$ , the shear stress acting on the interface of liquids has little effect on its shape. Also,  $We_{flow} \in (10^{-4}, 10^{-1})$  indicated regimes were dominated by

interfacial tension. The typical speed of the interface was calculated as  $u_{intfc} \approx l_{or}f$ . The characteristic length scale for evolution of the tip was on the order of  $w_{or}$ . These estimates were used to calculate  $Ca_{intfc} = \mu u_{intfc}/\gamma$  and  $We_{intfc} = \rho u_{intfc}^2 w_{or}/\gamma$  which obtained  $Ca_{intfc} \in (10^{-2}, 1)$  and  $We_{intfc} \in (10^{-4}, 1)$ . The values suggested that viscous and inertia forces were not dominated by capillary pressure. With increase in the flow rate, both viscous and inertial effects tended to play the dominant role. As a result, ( $Ca_{flow}$ ,  $We_{flow}$ ) and ( $Ca_{intfc}$ ,  $We_{intfc}$ ) showed interfacial tension dominated the dynamics of both droplets breakup and the relaxation of the interface. This is the so-called bifurcation under low flow rate, which is different from the dripping faucet (Ambravaneswaran et al. 2000) or the inverted dripping faucet (Garstecki et al. 2005a).

Compared LFF with SFF, it was found that the longer orifice induced higher frequency of droplets generation, which indicated faster interface velocity and higher  $We$ . In this sense, inertial term led to the bifurcation, just like the mechanism of inverted dripping faucet (Garstecki et al. 2005a). However, according to the non-dimensional parameter analysis, interfacial tension occupied the dominant position. As a result, the capillary pressure and inertia worked together to induce the secondary droplets under low flow rate within a certain range (low value of  $Q_c Q_d$  less than 0.5, Fig. 3b). As the flow rates continued to increase, shear stress was firstly dominated, which led to dripping regime again. Then, inertia and interfacial tension exceeded the other factors; nonlinear phenomenon appeared.

A novel secondary droplet phenomenon under low flow rate was studied in this communication. Unlike the traditional high-order nonlinear phenomenon, this regime presented period 1–2–1 transition. Flow-focusing droplet generator with long orifice tended to induce this transition, while the short orifice geometry was attributed to a different manner. Appearance of bifurcation under low flow rate was dominated by interfacial tension, and it was further strongly influenced by inertial term.

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## References

- Ambravaneswaran B, Phillips SD, Basaran OA (2000) Theoretical analysis of a dripping faucet. *Phys Rev Lett* 85:5332
- Anna SL, Bontoux N, Stone HA (2003) Formation of dispersions using “flow focusing” in microchannels. *Appl Phys Lett* 82:364–366

- Baah D, Floyd-Smith T (2014) Microfluidics for particle synthesis from photocrosslinkable materials. *Microfluid Nanofluid* 17:431–455
- Coullet P, Mahadevan L, Riera C (2004) Hydro-dynamical models for the chaotic dripping faucet. arXiv preprint physics/0408096
- El-Ali J, Sorger PK, Jensen KF (2006) Cells on chips. *Nature* 442:403–411
- Garstecki P, Fuerstman MJ, Whitesides GM (2005a) Nonlinear dynamics of a flow-focusing bubble generator: an inverted dripping faucet. *Phys Rev Lett* 94:234502
- Garstecki P, Stone HA, Whitesides GM (2005b) Mechanism for flow-rate controlled breakup in confined geometries: a route to monodisperse emulsions. *Phys Rev Lett* 94:164501
- Gulati S, Vijayakumar K, Good WW, Tamayo WL, Patel AR, Niu X (2016) Microdroplet formation in rounded flow-focusing junctions. *Microfluid Nanofluid* 20:2
- Gupta A, Matharoo HS, Makkar D, Kumar R (2014) Droplet formation via squeezing mechanism in a microfluidic flow-focusing device. *Comput Fluids* 4(100):218–226
- Lee W, Walker LM, Anna SL (2009) Role of geometry and fluid properties in droplet and thread formation processes in planar flow focusing. *Phys Fluids* 21:032103
- Liu M-Y, Hu Z-D, Li J-H (2004) Multi-scale characteristics of chaos behavior in gas-liquid bubble columns. *Chem Eng Commun* 191:1003–1016
- Maan AA, Nazir A, Khan MKI, Boom R, Schroën K (2015) Microfluidic emulsification in food processing. *J Food Eng* 147:1–7
- Nie Z, Seo M, Xu S, Lewis PC, Mok M, Kumacheva E, Whitesides GM, Garstecki P, Stone HA (2008) Emulsification in a microfluidic flow-focusing device: effect of the viscosities of the liquids. *Microfluid Nanofluid* 5:585–594
- Tan Y-C, Cristini V, Lee AP (2006) Monodispersed microfluidic droplet generation by shear focusing microfluidic device. *Sens Actuators B* 114:350–356
- Van Wamel A, Healey A, Sontum PC, Kvåle S, Bush N, Bamber J, de Lange Davies C (2016) Acoustic cluster therapy (ACT)—pre-clinical proof of principle for local drug delivery and enhanced uptake. *J Control Release* 224:158–164
- Woodward A, Cosgrove T, Espidel J, Jenkins P, Shaw N (2007) Monodisperse emulsions from a microfluidic device, characterised by diffusion NMR. *Soft Matter* 3:627–633
- Xu Q, Nakajima M (2004) The generation of highly monodisperse droplets through the breakup of hydrodynamically focused microthread in a microfluidic device. *Appl Phys Lett* 85:3726–3728