

# Journal of Intelligent Material Systems and Structures

<http://jim.sagepub.com/>

---

## **Design and analyses of axial semi-active dynamic vibration absorbers based on magnetorheological elastomers**

Zhirong Yang, Chunyun Qin, Zhushi Rao, Na Ta and Xinglong Gong

*Journal of Intelligent Material Systems and Structures* 2014 25: 2199 originally published online 13 January 2014

DOI: 10.1177/1045389X13519002

The online version of this article can be found at:

<http://jim.sagepub.com/content/25/17/2199>

---

Published by:



<http://www.sagepublications.com>

Additional services and information for *Journal of Intelligent Material Systems and Structures* can be found at:

**Email Alerts:** <http://jim.sagepub.com/cgi/alerts>

**Subscriptions:** <http://jim.sagepub.com/subscriptions>

**Reprints:** <http://www.sagepub.com/journalsReprints.nav>

**Permissions:** <http://www.sagepub.com/journalsPermissions.nav>


**Citations:** <http://jim.sagepub.com/content/25/17/2199.refs.html>

>> [Version of Record](#) - Oct 10, 2014

[OnlineFirst Version of Record](#) - Jan 13, 2014

[What is This?](#)

# Design and analyses of axial semi-active dynamic vibration absorbers based on magnetorheological elastomers

*Journal of Intelligent Material Systems and Structures*  
2014, Vol. 25(17) 2199–2207  
© The Author(s) 2014  
Reprints and permissions:  
sagepub.co.uk/journalsPermissions.nav  
DOI: 10.1177/1045389X13519002  
jim.sagepub.com  


Zhirong Yang<sup>1,2</sup>, Chunyun Qin<sup>1</sup>, Zhushi Rao<sup>1</sup>, Na Ta<sup>1</sup> and Xinglong Gong<sup>3</sup>

## Abstract

Magnetorheological elastomers are a new kind of magnetorheological materials mainly composed of polymer rubber and micro-sized magnetizable iron particles. Dynamic vibration absorbers based on magnetorheological elastomers are widely used in vibration systems with small amplitude since they have the advantages of no sealing equipments, good stability, and rapid response. In this article, a shear-mode semi-active dynamic vibration absorber based on magnetorheological elastomers is proposed, and the dynamic design principle of an axial semi-active dynamic vibration absorber attached to ship shafting is studied. The material preparation of magnetorheological elastomers and their properties under different magnetic fields are discussed. The structure of a single semi-active dynamic vibration absorber is designed, and theoretical analysis of shift-frequency property of a single semi-active dynamic vibration absorber is also investigated. The magnetic flux density of magnetorheological elastomers in the semi-active dynamic vibration absorber is analyzed using ANSYS software. A compact and efficient semi-active dynamic vibration absorber for shafting axial vibration control is proposed. Furthermore, a linear relationship is found between the excitation current and the natural frequency of a single semi-active dynamic vibration absorber. The results show that the designed axial semi-active dynamic vibration absorbers have better performance than classic passive dynamic vibration absorbers in terms of frequency-shift property and vibration absorption capacity.

## Keywords

Magnetorheological elastomers, intelligent materials, semi-active dynamic vibration absorber, ship shafting axial vibration, finite element method

## Introduction

The propeller is the main propulsion of ships and underwater vehicles, which works in spatially nonuniform wake field wherein axial pulse forces are produced and transferred to the hull through the propulsion shaft, thrust bearing, and bases. Consequently, axial vibration of shafting occurs and propagates through the supporting structure to the hull, where the vibration is radiated as structure-borne noise (Dylejko et al., 2007). In a series of means to reduce the axial vibration of shafting, the installation of a dynamic vibration absorber (DVA) is found to be an effective and feasible method. Many types of vibration absorbers exist, including classic passive, active, and semi-active absorbers. Since classic passive absorber operates at a single excitation frequency, resonances in the system appear just above and/or below the excitation frequency, making the effective bandwidth of passive absorber very narrow and becoming inefficient as the excitation

frequency shifts (Holdhusen, 2005). To overcome these disadvantages and extend the effective bandwidth of passive absorbers, active and semi-active absorbers are developed and introduced. The active dynamic vibration absorber (ADVA) is achieved using external actuator forces to offset the excitation forces and suppress the vibration (Okada and Okashita, 1990). The main

<sup>1</sup>Institute of Vibration, Shock and Noise, State Key Laboratory of Mechanical System and Vibration, Shanghai Jiaotong University, Shanghai, China

<sup>2</sup>Marine Engineering Institute, Jimei University, Xiamen, China

<sup>3</sup>CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Mechanics and Mechanical Engineering, University of Science and Technology of China, Hefei, China

## Corresponding author:

Zhushi Rao, Institute of Vibration, Shock and Noise, State Key Laboratory of Mechanical System and Vibration, Shanghai Jiaotong University, Shanghai 200240, China.  
Email: zsrhao@sjtu.edu.cn

advantage of the active dynamic absorber is that it enables fast response to disturbances. Some disadvantages of the active dynamic absorber are the potential use for large actuator forces, which may require high power inputs, increasing complexity of the system and the possible instability particularly when the excitation frequency moves away from the natural frequency of the system (Davis and Lesieutre, 2000). Semi-active dynamic vibration absorber (SDVA) controls the structural vibration by changing its dynamic parameters, such as the stiffness or damping. Some advantages of semi-active control are that it requires less energy, which lowers the costs, and its complexity is reduced in comparison with active systems, while being nearly as effective. There exist two major categories of variable stiffness elements, namely, mechanically variable spring and intelligent materials. Walsh and Lamancusa (1992) reduced transient vibrations using a mechanically variable spring but found disadvantage of long response time of the mechanical structure and difficulty in meeting the design requirements of the system, which rapidly changes the vibration characteristics. Intelligent materials can also be used as variable stiffness elements such as piezoelectric ceramics and magnetorheological elastomers (MREs). Piezoelectric elements have relatively high stiffness that can lead to relatively large absorber masses. Also, piezoelectric materials should not be put into tension, as they are fragile and have displacement amplitude limitations. To make the device more robust and reduce the mass of absorber, MREs can be used as variable stiffness elements (Ginder et al., 2000; Holdhusen, 2005).

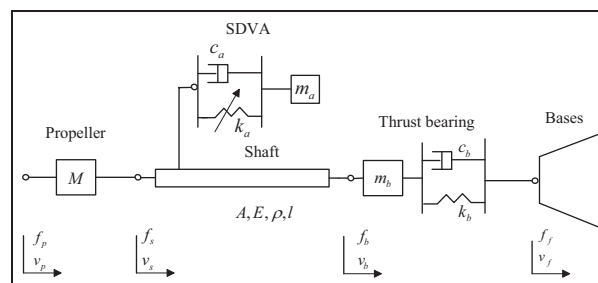
MREs are composites whose highly elastic polymer matrices are filled with magnetic particles. Typically, magnetic fields are applied to the polymer composite during cross-linking so that chain-like structures can be formed and fixed in the matrix after curing. The unique characteristic of MRE is that its shear storage modulus can be controlled by the external magnetic field rapidly, continuously, and reversibly (Deng and Gong, 2007). Such properties make MREs promising in many applications, such as SDVAs, stiffness tunable mounts and suspensions, and variable impedance surfaces. Watson (1997) applied a patent using MREs for a suspension bushing. Ginder et al. (2001) developed an adaptive tunable vibration absorber using MREs. Hoang et al. (2009, 2013) developed a torsional adaptive tunable vibration absorber using an MRE for vibration reduction of a powertrain test rig; the experimental results show that the ADVA can work in a frequency range from 10.75 to 16.5 Hz (53% relative change), and the numerical simulations show that the steady-state vibration of powertrain can be significantly reduced. Kim et al. (2011) proposed a real-time control system for the MRE-based DVA to suppress the cryogenic cooler vibrations. Research has also been conducted on optimizing the tuning parameters of vibration absorbers

applied to continuous systems. Jacquot (1978) modeled a continuous beam as a 1-degree-of-freedom system and determined the optimum absorber parameters for this equivalent lumped mass system.

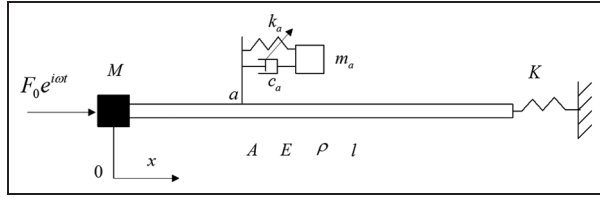
This study aims to investigate the application of MREs in axial SDVAs for reducing axial vibration of ship shafting. A compact and utilizable shear-mode vibration absorber is developed. The dynamic design principle of an axial SDVA attached to ship shafting, which is modeled as a continuous beam system, is studied. In addition, the material preparation of MREs and their properties under different magnetic fields are discussed. The structure of a single SDVA that can be used for ship shafting axial vibration control is designed and theoretical analysis of shift-frequency property of a single SDVA is also investigated. The magnetic flux density of MREs in the SDVA is analyzed using ANSYS software. Furthermore, the relationship between the excitation current and the natural frequency of a single SDVA is determined. Finally, a finite element method (FEM) structural model of ship shafting with SDVAs is established, and the dynamic responses of ship shafting–SDVA system are calculated to estimate the vibration absorption capability of SDVAs compared with the classic passive DVAs and ship shafting without DVAs.

## Design principle of SDVA

A typical ship shafting system with SDVA is illustrated in Figure 1. As shown in this figure, the propulsion shafting mainly consists of a propeller, propeller shaft, intermediate shaft, flange, thrust shaft, thrust bearing, and its bases. The propeller and its attached water are assumed as a lumped mass  $M$  while the propulsion shaft is modeled as a distributed mass and flexibility homogeneous beam. The thrust bearing and the bases are modeled by linear spring with a stiffness  $K$  taken in the longitudinal direction. The SDVA is simplified as mass–spring–damper system, in which the  $m_a$ ,  $k_a$ , and  $c_a$  denote the mass, stiffness, and damping coefficient, respectively. The simplified model of ship shafting–SDVA system is shown in Figure 2. An assumed mode method is employed to derive the discretized equations



**Figure 1.** Installation of SDVA on ship shafting. SDVA: semi-active dynamic vibration absorber.



**Figure 2.** Simplified model of ship shafting-SDVA system.  
SDVA: semi-active dynamic vibration absorber.

of motion. The axial displacement field of the shaft is expanded in the following form

$$u(x, t) = \sum_{i=1}^N \varphi_i(x) q_i(t) \quad (1)$$

where  $N$  is the maximum number of modes taken for shaft.  $\varphi_i$  is the  $i$ th mode function and  $q$  denotes the corresponding generalized displacement.

The total kinetic energy of the system, including the energy contributions of the ship shafting and the attached absorbers, is given as

$$T = \frac{1}{2} m_a \dot{u}_a^2 + \frac{1}{2} \int_0^l \rho A(x) \dot{u}^2(x, t) dx + \frac{1}{2} M \cdot \dot{u}^2(0, t) \quad (2)$$

where  $\rho$  and  $A(x)$  represent the mass density and the cross-sectional area of the shaft, respectively;  $M$  is the lumped mass of propeller;  $m_a$  is the mass of the attached absorber; and  $\dot{u}_a$  and  $\dot{u}(0, t)$  are the axial velocities of absorber and the shaft at  $x = 0$ , respectively.

The potential energy of the shaft and springs due to deformation can be expressed as

$$U = \frac{1}{2} k_a [u(a, t) - u_a]^2 + \frac{1}{2} \int_0^l EA(x) \left( \frac{\partial u(x, t)}{\partial x} \right)^2 dx + \frac{1}{2} K \cdot u^2(l, t) \quad (3)$$

where  $E$  is the modulus of elasticity;  $K$  is the longitudinal stiffness of thrust bearing;  $k_a$  is the absorber's stiffness;  $u(l, t)$  and  $u(a, t)$  denote the axial displacements of the shaft at the location of  $x = l$  and  $x = a$ , respectively; and  $u_a$  is the axial displacement of the absorber.

Generally, the propeller forces mainly excite the first-order vibration mode of the ship shafting system with the normal working speed. Consequently, only the first-order vibration mode of the shaft is employed in this study. In doing so, equation (1) can be rewritten as

$$u(x, t) = \varphi_1(x) q_1(t) \quad (4)$$

Substituting equation (4) into equations (2) and (3), the equations of motion can be obtained using the Lagrange method. Considering the harmonic

concentrate excitation force  $F_0 e^{i\omega t}$  applied at location  $x = 0$  on the shaft, the equations of motion of shafting-absorber system can be written in matrix form as follows

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_a \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{u}_a(t) \end{Bmatrix} + \begin{bmatrix} k_1 + k_a \varphi_1^2(a) & -k_a \varphi_1(a) \\ -k_a \varphi_1(a) & k_a \end{bmatrix} \begin{Bmatrix} q_1(t) \\ u_a(t) \end{Bmatrix} = \begin{bmatrix} F_0 \varphi_1(0) \\ 0 \end{bmatrix} e^{i\omega t} \quad (5)$$

where  $m_1 = M \varphi_1^2(0) + \rho A l$  and  $k_1 = K \varphi_1^2(l) + \int_0^l EA(x) (\partial \varphi_1(x) / \partial x)^2 dx$ .

The solution to equation (5) for the generalized coordinate  $q_1(t)$  is given as

$$q_1(t) = \frac{(k_a - m_a \omega^2) F_0 \varphi_1(0) e^{i\omega t}}{m_1 m_a \omega^4 - \omega^2 [m_1 k_a + m_a k_a \varphi_1^2(a) + m_a k_1] + k_a k_1} \quad (6)$$

For simplicity in the mathematical formulation, the following dimensionless parameters are introduced

$$\delta_{st} = \frac{F_0}{K}, \quad \mu = \frac{m_a}{m_1}, \quad \omega_n^2 = \frac{k_1}{m_1}, \quad \omega_a^2 = \frac{k_a}{m_a}, \quad f = \frac{\omega}{\omega_a}, \quad g = \frac{\omega}{\omega_n} \quad (7)$$

where  $K$  is the longitudinal stiffness of thrust bearing and  $\delta_{st}$  is the static deformation.

Substituting equation (7) into equation (6), one obtains

$$\frac{q_1}{\delta_{st}} = \frac{(1 - f^2)}{f^2 g^2 - f^2 - [1 + \mu \varphi_1^2(a)] g^2 + 1} \cdot \frac{K \varphi_1(0)}{k_1} e^{i\omega t} \quad (8)$$

It can be seen from equation (8) that the response of isolated shaft is minimum ( $q_1/\delta_{st} = 0$ ) when  $f$  is taken to be 1. In other words, when the natural frequency of dynamic absorber is equal to excitation frequency of external force ( $\omega_a = \omega$ ), the vibration absorption capability of the absorber is maximum. The SDVA introduced in this article is based on this dynamic principle, which can vary its natural frequency as the excitation frequency is changed, to make sure the natural frequency of dynamic absorber traces the excitation frequency of external force on time.

Considering the damping  $c_a$  of the absorber, we set the absorber's damping ratio as  $\zeta = c_a / 2m_a \omega_a$ . Equation (8) becomes

$$\left| \frac{q_1(t)}{\delta_{st}} \right| = \sqrt{\frac{(1 - f^2)^2 + (2\zeta f)^2}{X^2 + \Delta^2}} \cdot \left| \frac{K \varphi_1(0)}{k_1} \right| \quad (9)$$

where  $X = f^2 g^2 - f^2 - [1 + \mu \varphi_1^2(a)] g^2 + 1$  and  $\Delta = 2\zeta f [(1 - g^2) - \mu \varphi_1^2(a) g^2]$ .





**Figure 3.** MRE sample.  
MRE: magnetorheological elastomer.

### Material preparation of MREs and their properties

The sample ingredients of MREs are natural rubber as a matrix, plasticizer, additives, and carbonyl iron particles with an average diameter of  $3\ \mu\text{m}$  (Gong et al., 2005). At the first step of the fabricating process, all ingredients are thoroughly mixed by mixing roll and then the mixture is compressed into a mold and placed in a self-developed magnet-heat coupled device. During the pre-cured stage, the particles are magnetized and form chains aligned along the magnetic field direction. Thirty minutes later, the magnetic field is turned off and the temperature is raised to  $153^\circ\text{C}$  to finish the sulfuration (Deng and Gong, 2007). The fabricated sample, with diameter of 50 mm and thickness of 5 mm, is shown in Figure 3.

The material properties of a sample under various magnetic fields are evaluated by a modified dynamic mechanical analyzer (DMA) from Triton Co., as

shown in Figure 4. To measure the properties of MRE sample, such as shear storage modulus and loss factor, under various magnetic fields, an electromagnet is developed, which can provide magnetic flux density up to 1100 mT. In the experiments, the sample is cut into cuboids of  $10\ \text{mm} \times 10\ \text{mm} \times 5\ \text{mm}$ . The range of the external magnetic flux density is 0–1000 mT, the driving frequency is fixed at 10 Hz, and the dynamic strain amplitude is set as 0.1%. The experiments are carried out at room temperature. The results are shown in Figures 5 and 6. It can be seen that the zero-field shear storage modulus is  $G_0 = 0.74\ \text{MPa}$ , the magneto-induced shear storage modulus is  $\Delta G = 3.23\ \text{MPa}$ , and the magneto-induced change in the loss factor is 0.11. It can be seen from Figure 5 that the shear storage modulus shows an increasing trend with magnetic flux density. However, the increasing slope decreases with the increment of magnetic flux density, which is due to the magnetic saturation. Figure 6 shows the loss factor is independent of magnetic flux density. To facilitate the SDVA design and simulation analysis, the equation of the shear storage modulus of MREs is derived from Figure 5 as follows

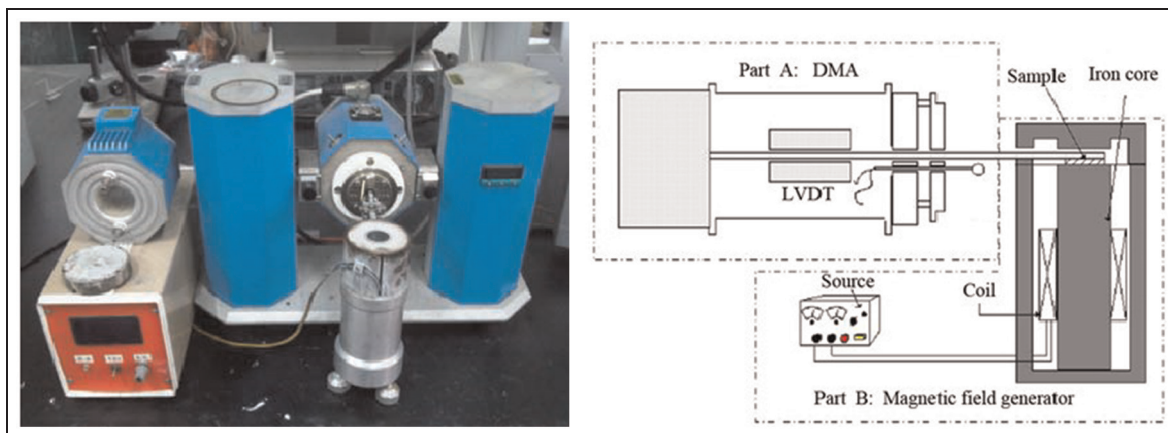
$$G = \begin{cases} 10.68B^2 + 0.7729B + 0.7323 & 0 \leq B < 0.3\ \text{T} \\ 5.512(B - 0.3) + 1.923 & 0.3\ \text{T} \leq B < 0.6\ \text{T} \\ -4.56B^2 + 8.167B + 0.3339 & 0.6\ \text{T} \leq B < 1\ \text{T} \end{cases} \quad (10)$$

The equation will be used to design the single SDVA in the following section.

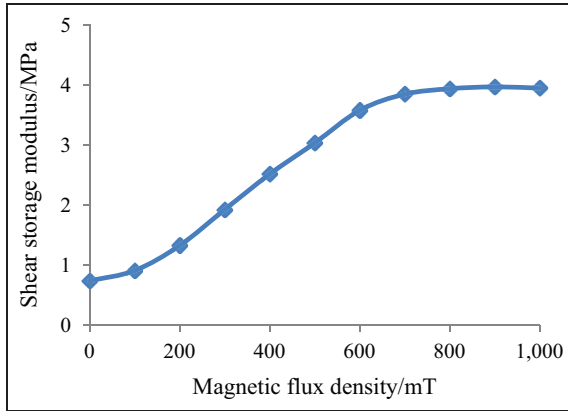
### Design and analyses of a single SDVA

#### Structure of a single SDVA

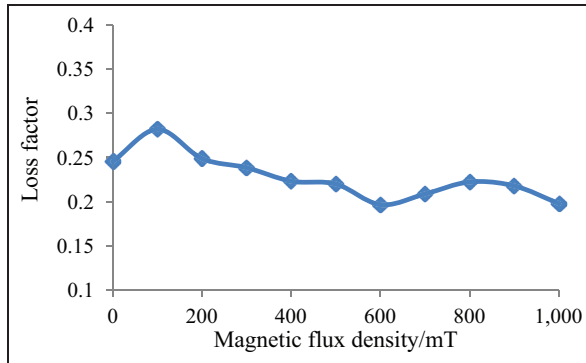
The schematic diagram of the MRE-based SDVA is shown in Figure 7. As shown in this figure, the MRE-based SDVA consists of three main components: the oscillator or dynamic mass, smart spring elements with



**Figure 4.** DMA measurement system.  
DMA: dynamic mechanical analyzer; LVDT: linear variable displacement transducer.



**Figure 5.** Shear storage modulus versus magnetic flux density.



**Figure 6.** Loss factor versus magnetic flux density.

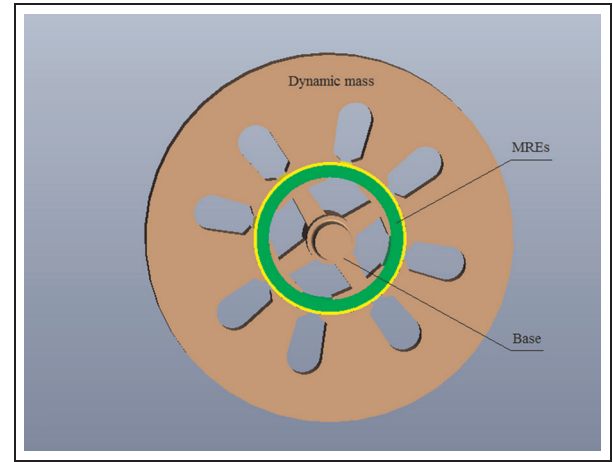
MREs, and the base attached to the shafting indirectly. The smart spring elements (MREs) connect the dynamic mass and the base. The dynamic mass and the base are made of low-carbon steel. The current coils are strapped on the base to emit the magnetic field acting on MREs and some grooves are slotted in the dynamic mass to install the current coils to increase the magnetic field intensity. The sketch of the distribution of current coils and magnetic circuit among SDVA can be seen from Figure 8. It can be seen that the dynamic mass, MREs, and the base form the closed circuit. This development makes SDVA more compact and more efficient for shafting axial vibration control as no additional oscillators and complex mechanical devices are required.

### Theoretical analysis of shift-frequency property

The shear storage modulus  $G$  of MREs consists of two terms: zero-field shear storage modulus  $G_0$  and magneto-induced shear storage modulus  $\Delta G$

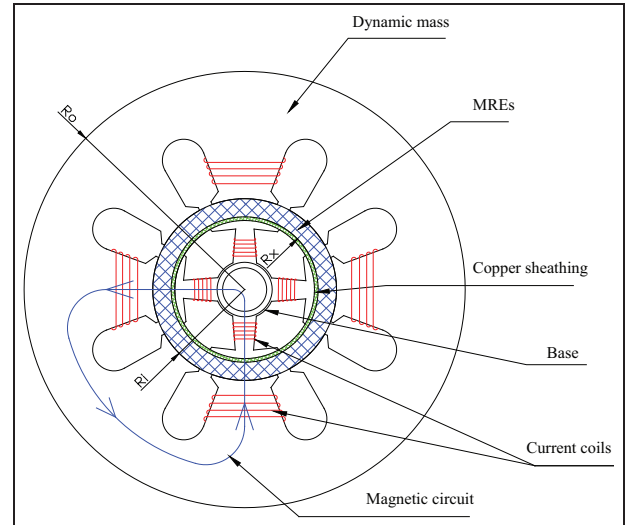
$$G = G_0 + \Delta G \quad (11)$$

The natural frequency of a single SDVA can be expressed as



**Figure 7.** Structure of a single SDVA.

SDVA: semi-active dynamic vibration absorber; MRE: magnetorheological elastomer.



**Figure 8.** Sketch of the distribution of current coils and magnetic circuit among SDVA.

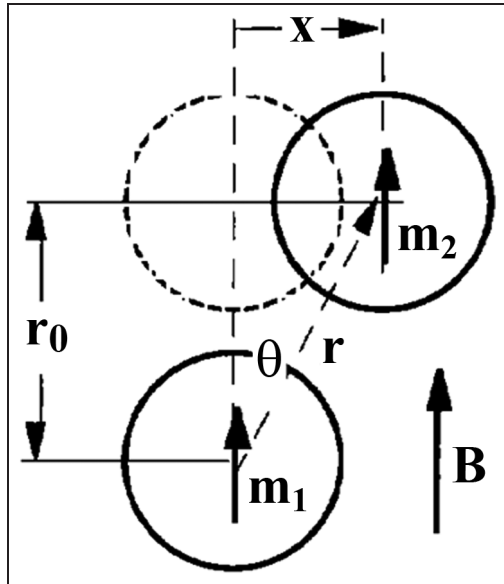
SDVA: semi-active dynamic vibration absorber; MRE: magnetorheological elastomer.

$$\hat{f} = \hat{f}_0 + \Delta \hat{f} \quad (12)$$

where  $\hat{f}_0$  is the initial natural frequency and  $\Delta \hat{f}$  is the magneto-induced frequency

$$\begin{aligned} \hat{f}_0 &= \frac{1}{2\pi} \sqrt{\frac{G_0 A}{m_a h}} = \frac{1}{2\pi} \sqrt{\frac{G_0 \pi (R_i + R_x) l_s}{\rho (R_i - R_x) \pi (R_o^2 - R_i^2) l_s}} \\ &= \frac{1}{2\pi} \sqrt{\frac{G_0 (R_i + R_x)}{\rho (R_i - R_x) (R_o^2 - R_i^2)}} \end{aligned} \quad (13)$$

$$\Delta \hat{f} = \frac{1}{2\pi} \sqrt{\frac{G_0 (R_i + R_x)}{\rho (R_i - R_x) (R_o^2 - R_i^2)}} \left( \sqrt{1 + \frac{\Delta G}{G_0}} - 1 \right) \quad (14)$$



**Figure 9.** Magnetic interaction of the two particles modeled as dipole moments  $\vec{m}$  and sheared with respect to one another.

where  $R_o$  and  $R_i$  are the outer radius and inner radius of dynamic mass, respectively;  $R_x$  is the inner radius of MREs; and  $l_s$  is the length of SDVA, as shown in Figure 8.

From equation (13), the initial natural frequency of the SDVA can be designed to match the primary system. Equation (14) reveals that the frequency-shift capacity is not only related to the magnetorheological effect but also related to the initial shear storage modulus. Large initial modulus with the same magnetorheological effect will cause wide frequency-shift bandwidth. To theoretically analyze the frequency-shift property with magnetic field, a magnetic dipole model considering the interaction between all particles at the same chain is employed (Jolly et al., 1996). Figure 9 shows two adjacent particles (dipoles) in a chain along with the direction of applied magnetic field. The interaction energy of the two dipoles of equal strength  $|m|$  and direction is

$$E_{12} = \frac{1}{4\pi\mu_0\mu_1} \left[ \frac{\vec{m}_1 \cdot \vec{m}_2}{r^3} - \frac{3(\vec{m}_1 \cdot \vec{r})(\vec{m}_2 \cdot \vec{r})}{r^5} \right] \quad (15)$$

$$= \frac{|m|^2(1 - 3\cos^2\theta)}{4\pi\mu_0\mu_1|r|^3} = \frac{|m|^2 \left(1 - 3\frac{r_0^2}{r_0^2 + x^2}\right)}{4\pi\mu_0\mu_1(r_0^2 + x^2)^{3/2}}$$

where  $\mu_1$  is the relative permeability of MREs,  $\vec{m}$  is the dipole moment of each particle, and some basic trigonometric identities have been used in the second equality. By defining the scalar shear strain of the particle chain as  $\varepsilon = x/r_0$ , the interaction energy can be written as

$$E_{12} = \frac{|m|^2(\varepsilon^2 - 2)}{4\pi\mu_0\mu_1 r_0^3(\varepsilon^2 + 1)^{5/2}} \quad (16)$$

Now, it is assumed that the particles are aligned in long chains, and that there is only magnetic interaction between adjacent particles within the chain, that is, there are no multipole interactions. For a dipole  $i$ , the dipole moment  $\vec{m}_i$  is determined by

$$\vec{m}_i = \frac{1}{6} \pi d^3 \mu_0 \mu_1 \chi H_i \quad (17)$$

where  $d$  is the particle diameter,  $\chi$  is the susceptibility of iron particles,  $H_i$  is the magnetic field strength at this point,  $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$ , and  $\mu_1$  is the permeability of MREs. The field experienced by a dipolar particle  $i$  is the resultant of an applied external field  $H_0$  and ignores the local field induced by the particles around the particle  $i$ .

The total energy density (energy per unit volume) associated with the one-dimensional shear strain can be calculated by multiplying the particle-to-particle energy, given by equation (16), by the total number of particles and dividing by the total volume

$$U = \frac{n \cdot E_{12}}{V_e} = \frac{n \cdot V_i \cdot E_{12}}{V_e \cdot V_i} = \frac{\phi \cdot E_{12}}{V_i} \quad (18)$$

$$= \frac{3\phi(\varepsilon^2 - 2)|m|^2}{2\pi^2\mu_0\mu_1 d^3 r_0^3 (\varepsilon^2 + 1)^{5/2}}$$

where  $V_e$  is the total volume of MREs,  $V_i$  is the volume of each particle,  $n$  is the total number of particles, and  $\phi$  is the volume fraction of particles in the composite. The stress induced by the application of a magnetic field can be computed by taking the derivative of inter-particle energy density with respect to the scalar shear strain

$$\sigma = \frac{\partial U}{\partial \varepsilon} = \frac{9\phi\varepsilon(4 - \varepsilon^2)|m|^2}{2\pi^2\mu_0\mu_1 d^3 r_0^3 (\varepsilon^2 + 1)^{7/2}} \quad (19)$$

If equation (17) is substituted into equation (19) and the derivative of the stress is further taken with respect to the scalar shear strain, the magneto-induced shear storage modulus can be written as

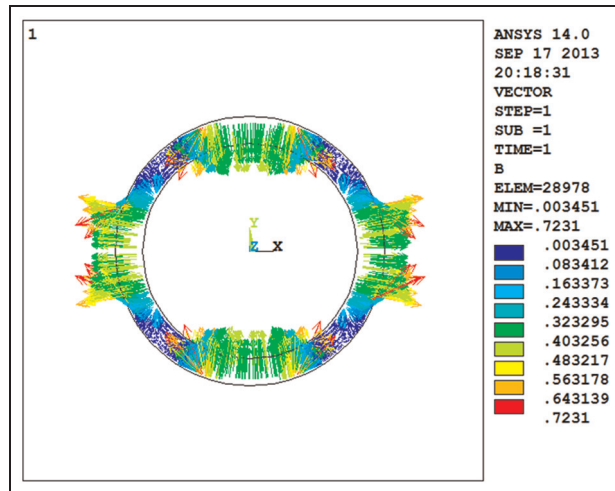
$$\Delta G = \frac{\partial \sigma}{\partial \varepsilon} = \frac{\phi\mu_0\mu_1(4\varepsilon^4 - 27\varepsilon^2 + 4)\chi^2 H_0^2}{8h_d^3(\varepsilon^2 + 1)^{9/2}} \quad (20)$$

where  $h_d = r_0/d$  is the indication of the gap between two adjacent particles in a chain. When the shear strain is small, equation (20) can be simplified as

$$\Delta G \approx \frac{\phi\mu_0\mu_1\chi^2 H_0^2}{2h_d^3} \quad (21)$$

Substituting equation (21) into equation (14), equation (14) can be rewritten as

$$\Delta \hat{f} = \frac{1}{2\pi} \sqrt{\frac{G_0(R_i + R_x)}{\rho(R_i - R_x)(R_o^2 - R_i^2)}} \left( \sqrt{1 + \frac{\phi\mu_0\mu_1\chi^2 H_0^2}{2h_d^3 G_0}} - 1 \right) \quad (22)$$



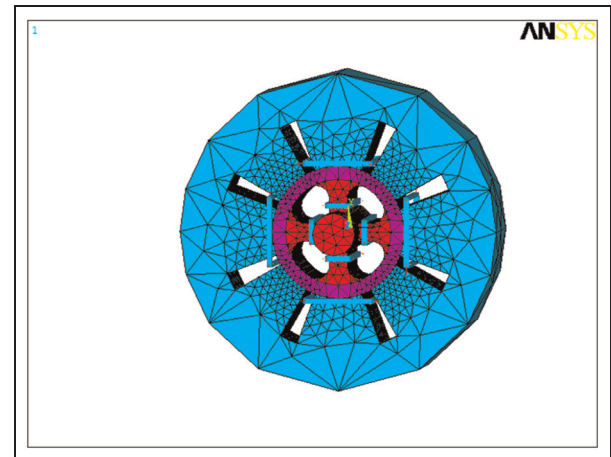
**Figure 10.** Magnetic flux density of MREs in the single SDVA. MRE: magnetorheological elastomer; SDVA: semi-active dynamic vibration absorber.

It can be seen from equation (21) that the magneto-induced shear storage modulus rises quadratically with the applied external field  $H_0$ . In addition, the change in shear storage modulus of MREs is greatly affected by the ratio of mean distance between two adjacent particles to the mean radius of the particles. It can be seen from equation (22) that it should have a relatively high particle volume fraction of MREs, increase the applied external field, reduce the gap between two adjacent particles in a chain, and have the high-saturated magnetic flux density of iron particles to obtain a large shift of natural frequency of SDVA.

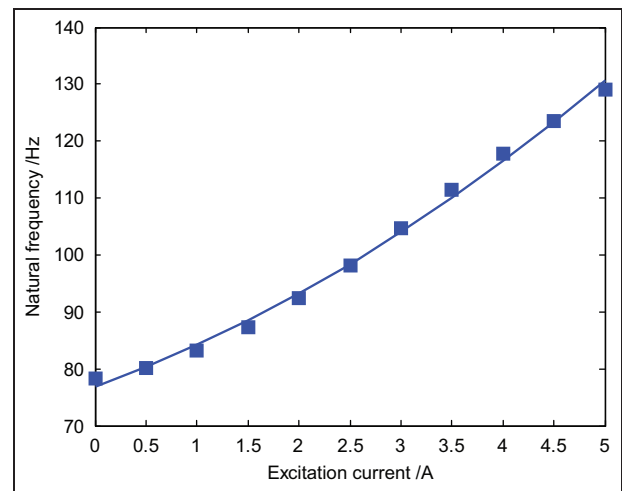
### Electromagnetic and modal analyses

In order to optimize the turn number of coils, the electromagnetic analysis of a single SDVA is simulated using the ANSYS software. The outer radius and inner radius of dynamic mass are  $R_o = 60$  mm and  $R_i = 25$  mm, respectively; the inner radius of MREs is  $R_x = 20$  mm; and the length of SDVA is  $l_s = 30$  mm. The relative permeability of the low-carbon steel and air is  $\mu_s = 2000$  and  $\mu_a = 1$ , respectively. The relative permeability of MREs is  $\mu_{MRE} = 2.91$  taken from the properties test of the sample. Figure 10 shows the magnetic flux density of MREs in the single SDVA while the coils are 230 rounds and the excitation current is 5 A. It can be seen that the averaged value of the magnetic flux density of MREs is about 400 mT, which is quite enough to meet the design requirements.

To investigate the frequency-shift capability of SDVA and find the relationship between the excitation current and the natural frequency of absorber, the modal analysis of SDVA under different excitation currents is also carried out. First, an FEM model of electromagnetic analysis of SDVA (as shown in Figure 11)



**Figure 11.** FEM model of magnetic analysis of SDVA. FEM: finite element method; SDVA: semi-active dynamic vibration absorber.



**Figure 12.** Relationship between the excitation current and the natural frequency of SDVA. SDVA: semi-active dynamic vibration absorber.

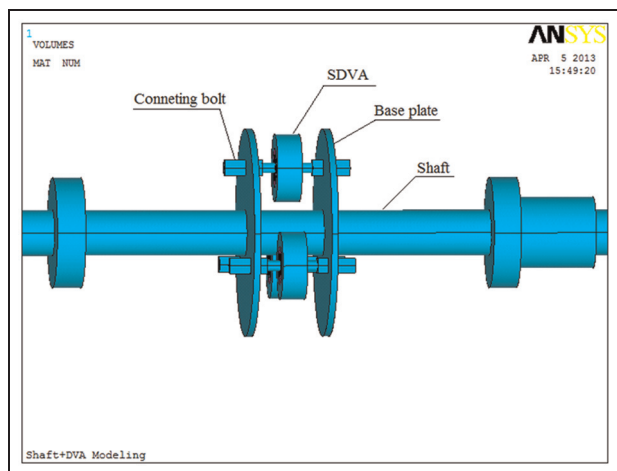
is developed and the magnetic flux density of every element in the model under a certain excitation current is calculated. Second, according to the variation in shear storage modulus with different magnetic flux densities mentioned in Figure 5 and equation (10), the shear storage modulus of every MRE material element with certain magnetic flux density can be obtained. Third, the FEM model of structural analysis of SDVA with the renewed shear storage modulus of every MRE material element is rebuilt and the modal analysis of SDVA under a certain excitation current is done to obtain the natural frequency of SDVA. Finally, the certain excitation current is changed and the above steps are repeated. The results are shown in Table 1 and Figure 12. It can be seen that the natural frequency of SDVA has an increasing trend with the excitation



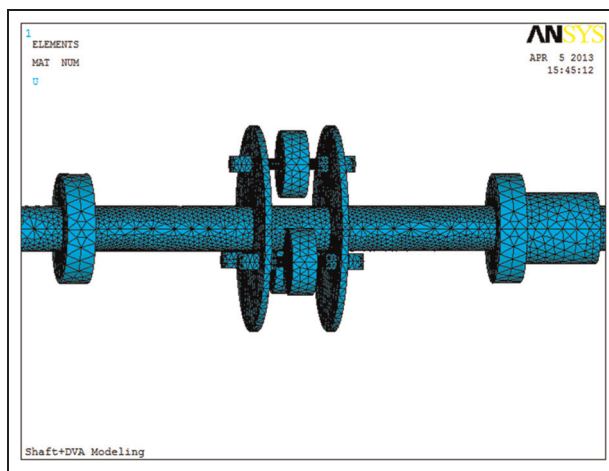
**Table 1.** Relationship between the excitation current and the natural frequency of SDVA.

Current (A)	0	0.5	1	1.5	2	2.5
Frequency (Hz)	78.20	80.11	83.20	87.35	92.41	98.22
Current (A)	3	3.5	4	4.5	5	
Frequency (Hz)	104.61	114.45	117.85	123.54	128.98	

SDVA: semi-active dynamic vibration absorber.



**Figure 13.** Structure of ship shafting-SDVA system.  
SDVA: semi-active dynamic vibration absorber.



**Figure 14.** FEM model of ship shafting-SDVA system.  
FEM: finite element method; SDVA: semi-active dynamic vibration absorber.

current. The relationship between the excitation current and the natural frequency of SDVA is almost linear. The natural frequency increases from 78.20 Hz at 0 A to 128.89 Hz at 5 A and the relative frequency change is about 165%. Clearly, the magnetic flux density of the magnetic circuit is proportional to the excitation current. Therefore, the results' trend agrees well with equation (22) qualitatively. Using this relationship, the required excitation current is calculated for the excitation frequency and the magnetic field is controlled.

### Comparison of SDVAs and classic passive DVAs and without DVAs in the ship shafting

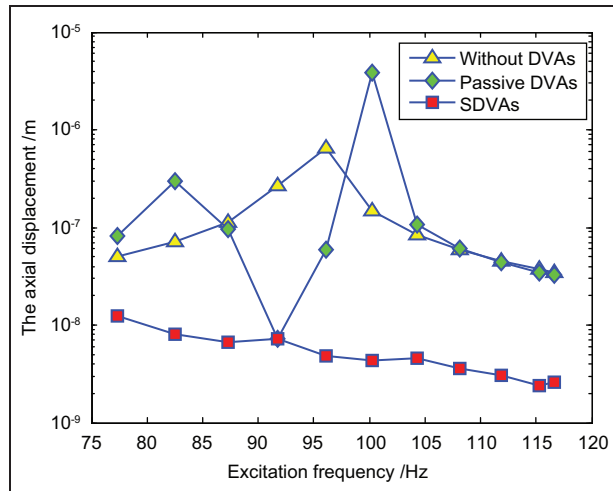
Since the mass of ship shafting is large, several SDVAs are linked in parallel and are connected to ship shafting to increase the mass ratio, but the stiffness of SDVAs remains the same as a single SDVA. Figure 13 shows that three SDVAs are linked in parallel to assemble the ship shafting-SDVA system. An FEM structural model of ship shafting with SDVAs is established using ANSYS software, as shown in Figure 14. The natural frequency of SDVAs is tuned to trace the excitation frequency of external force by adjusting the excitation current. The classic passive DVAs are realized by fixing the excitation current to ensure the fixed natural frequency of DVAs. The dynamic responses of ship

shafting with SDVAs and classic passive DVAs and without DVAs are all calculated. Furthermore, the axial displacements of the point on shaft, which is near to the thrust bearing, have been obtained. The vibration absorption capability of SDVAs is compared with the classic passive DVAs and ship shafting without DVAs, as shown in Figure 15.

It can be seen that the ship shafting without DVAs has the primary resonance peak of the shafting at 96 Hz and has the relatively large displacement responses in the whole excitation frequency range. For the ship shafting with classic passive DVAs, the best vibration attenuation efficiency occurs near to the natural frequency of the primary shafting system, but the effect becomes worse sharply while the excitation frequency is apart from the natural frequency and the new resonance peaks occur at 82.5 and 100 Hz, respectively. For the ship shafting with SDVAs, it has better vibration absorption capability than the classic passive DVAs in the whole excitation frequency bandwidth.

### Conclusion

In this study, a shear-mode SDVA based on MREs has been developed. The design principle of an axial SDVA attached to the ship shafting, which is modeled as a continuous beam system, is studied. In addition, the



**Figure 15.** Comparison of vibration attenuation of SDVAs and passive DVAs and without DVAs.

DVA: dynamic vibration absorber; SDVA: semi-active dynamic vibration absorber.

material preparation of MREs and their properties under different magnetic fields are discussed. The structure of a single SDVA is designed and theoretical analysis of shift-frequency property of a single SDVA is also investigated. The magnetic flux density of MREs in the SDVA is analyzed using ANSYS software. A compact and efficient SDVA for shafting axial vibration control is proposed. Furthermore, a linear relationship is found between the excitation current and the natural frequency of a single SDVA. The results show that the designed axial SDVAs have better performance than the classic passive DVAs in terms of frequency-shift property and vibration absorption capacity.

### Declaration of conflicting interests

The author declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

### Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

### References

Davis CL and Lesieutre GA (2000) An actively tuned solid-state vibration absorber using capacitive shunting of

- piezoelectric stiffness. *Journal of Sound and Vibration* 232: 601–617.
- Deng H and Gong X (2007) Adaptive tuned vibration absorber based on magnetorheological elastomer. *Journal of Intelligent Material Systems and Structures* 18: 1205–1210.
- Dylejko P, Kessissoglou N, Tso Y, et al. (2007) Optimisation of a resonance changer to minimise the vibration transmission in marine vessels. *Journal of Sound and Vibration* 300: 101–116.
- Ginder J, Nichols M, Elie L, et al. (2000) Controllable-stiffness components based on magnetorheological elastomers. *Proceedings of SPIE* 3985: 418–425.
- Ginder J, Schlotter W and Nichols M (2001) Magnetorheological elastomers in tunable vibration absorbers. *Proceedings of SPIE* 4331: 103–110.
- Gong X, Zhang X and Zhang P (2005) Fabrication and characterization of isotropic magnetorheological elastomers. *Polymer Testing* 24: 669–676.
- Hoang N, Zhang N and Du H (2009) A dynamic absorber with a soft magnetorheological elastomer for powertrain vibration suppression. *Smart Materials and Structures* 18: 074009 (10 pp.).
- Hoang N, Zhang N, Li W, et al. (2013) Development of a torsional dynamic absorber using a magnetorheological elastomer for vibration reduction of a powertrain test rig. *Journal of Intelligent Material Systems and Structures*. Epub ahead of print 8 May. DOI: 10.1177/1045389X13489361.
- Holdhusen MH (2005) *The state-switched absorber used for vibration control of continuous systems*. PhD Thesis, Georgia Institute of Technology, Atlanta, GA.
- Jacquot R (1978) Optimal dynamic vibration absorbers for general beam systems. *Journal of Sound and Vibration* 60: 535–542.
- Jolly MR, Carlson JD and Munoz BC (1996) A model of the behaviour of magnetorheological materials. *Smart Materials and Structures* 5: 607–614.
- Kim Y-K, Koo J-H, Kim K-S, et al. (2011) Developing a real time controlled adaptive MRE-based tunable vibration absorber system for a linear cryogenic cooler. In: *Proceedings of the IEEE/ASME international conference on Advanced Intelligent Mechatronics (AIM)*, Budapest, 3–7 July, pp. 287–290. New York: IEEE.
- Okada Y and Okashita R (1990) Adaptive control of an active mass damper to reduce structural vibration. *JSME International Journal Series III: Vibration, Control Engineering, Engineering for Industry* 33: 435–440.
- Walsh P and Lamancusa J (1992) A variable stiffness vibration absorber for minimization of transient vibrations. *Journal of Sound and Vibration* 158: 195–211.
- Watson J (1997) *Method and apparatus for varying the stiffness of a suspension bushing*. US Patent No. 5609353, USA.