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An effective permeability model to predict field-dependent modulus of magnetorheological elastomers

Xianzhou Zhang, Weihua Li *, X.L. Gong

School of Mechanical, Materials and Mechatronic Engineering, University of Wollongong, Wollongong, NSW 2522, Australia
CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Modern Mechanics, University of Science and Technology of China, Hefei 230027, China

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Abstract

Magnetorheological elastomers (MREs) are composites where magnetic particles are suspended in a non-magnetic solid or a gel-like matrix. MREs are shown to have a controllable, field-dependent shear modulus. Most of conventional MREs models are based on magnetic dipole interactions between two adjacent particles of the chain. These models can predict the field-dependent properties of MREs with simple chain-like structures. In this paper, an effective permeability model is proposed to predict the field-induced modulus of MREs. Based on the effective permeability rule and taking into account the particle's saturation, the model is proposed to predict the mechanical performances of MREs with complex structure and components. The effectiveness of the model is justified by a designed novel MREs.

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Keywords: Magnetorheological elastomers; Mechanical properties; Magnetic field; Field-dependent shear modulus

1. Introduction

Magnetorheological (MR) material is a class of smart materials whose rheological properties can be controlled rapidly and reversibly by the application of an external magnetic field. Traditionally, it is composed of MR fluids and MR foams, while MR elastomers become a new family member recently. MR materials typically consist of micron-sized magnetic particles suspended in a non-magnetic matrix. The magnetic interactions between particles in these composites depend on the magnetization orientation of each particle and on their spatial relationship, coupling the magnetic and strain fields in these materials and giving rise to a number of interesting magnetomechanical phenomena [1–6].

* Corresponding author. Tel.: +61 2 4221 3490; fax: +61 2 4221 3101.
E-mail address: weihuali@uow.edu.au (W. Li).

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MR elastomers (MREs) are composites where magnetic particles are suspended in a non-magnetic solid or a gel-like matrix. The particles inside the elastomer can be homogeneously distributed or they can be grouped (e.g. into chain-like columnar structures). To produce an aligned particle structure, the magnetic field is applied to the polymer composite during crosslinking so that the columnar structures can form and become locked in place upon the final cure. This kind of processing imparts special anisotropic properties to the viscoelastic materials. Only recently has the field responsiveness of the viscoelastic properties of these elastomers been explored [7–11].

MREs have a controllable, field-dependent modulus while MR fluids and MR foam have a field-dependent yield stress. This makes the two groups of materials complementary rather than competitive to each other. In other words, the strength of MR fluids is characterized by their field dependent yield stress while the strength of MREs is typically characterized by their field-dependent modulus. Other obvious advantages of MREs are that the particles are not able to settle with time and that there is no need for containers to keep the MR material in place. Because the chain-like or columnar structures have been locked in the rubber-like matrix during curing, the particles need no time to arrange again while MREs are applied an external magnetic field, thus the response time of MREs is much less than that of MR fluids (several ms).

MR fluids' field-dependent yield stress makes them to be widely used in various smart devices, such as dampers, clutches, and brakes [3,4]. There is little doubt that there are numerous applications that can make use of controllable stiffness and others unique characteristics of MREs, such as adaptive tuned vibration absorbers (TVAs), tuneable stiffness mounts and suspensions, and variable impedance surfaces.

Many models of MREs have been developed to predict MRE mechanical properties. Generally, mechanical properties of MREs can be divided into two distinctive regimes: the composite properties without a magnetic field and the composite properties with a magnetic field. Usually the host composite in MREs is a rubber-like material with a nonlinear stress–strain relationship. Ogden's model has been widely used to model rubber-like materials [8]. MREs modulus is also a function of filler (iron particles) volume fraction and their zero-field modulus can be given by Guth model [12]. Most models of MR material field-dependent behavior are based on the magnetic dipole interactions between two adjacent particles of the chain. Ginder and Davis [12,15] used finite element analysis method (FEM) to determine the values of the modulus under a varied magnetic field. For elastomer composites containing magnetically soft particles dispersed in natural rubber, a 40% maximum change in modulus was observed upon the application of a saturating magnetic field. The theoretical approaches show that the maximum shear modulus increment of conventional MREs is about 50% [12]. It is noted that these models are based on simple ball-chain structures without taking into account other complex structures or mixed components. Furthermore, the integradation between unsaturation and saturation of MREs is neglected too. In order to fabricate high quality MREs, the complex structure and components ought to be presented, and a new model of MREs must be used to explain the mechanical properties of them. This is the major motivation of the paper.

In this paper, a new model of MREs is proposed to explain field-induced modulus of MREs with complex structure and components and the particle's saturation is taken into account. A novel structure MREs is also introduced, as a case study, to evaluate the model.

2. Model of magnetic field induced increase in shear modulus

In this section, the conventional MREs with simple chain-like structure are presented to introduce a new effective permeability model at first. Then, the field-induced modulus of MREs with complex structure can be predicted by this model because the conventional magnetic dipole and correlative theory can not explain the field-induced modulus of such complex structure. Finally, the comparison with conventional and novel MREs' field-dependent modulus is given.

For structural constructions as shown in Fig. 1b, by using the Maxwell Garnett mixing rule [13], the effective permeability of chains can be predicted as

$$\mu_{\mathrm{ceff}} = \mu_{\mathrm{m}} + 2\phi\mu_{\mathrm{m}} \frac{\mu_{\mathrm{p}} - \mu_{\mathrm{m}}}{\mu_{\mathrm{p}} + \mu_{\mathrm{m}} - \phi(\mu_{\mathrm{p}} - \mu_{\mathrm{m}})} \tag{1}$$

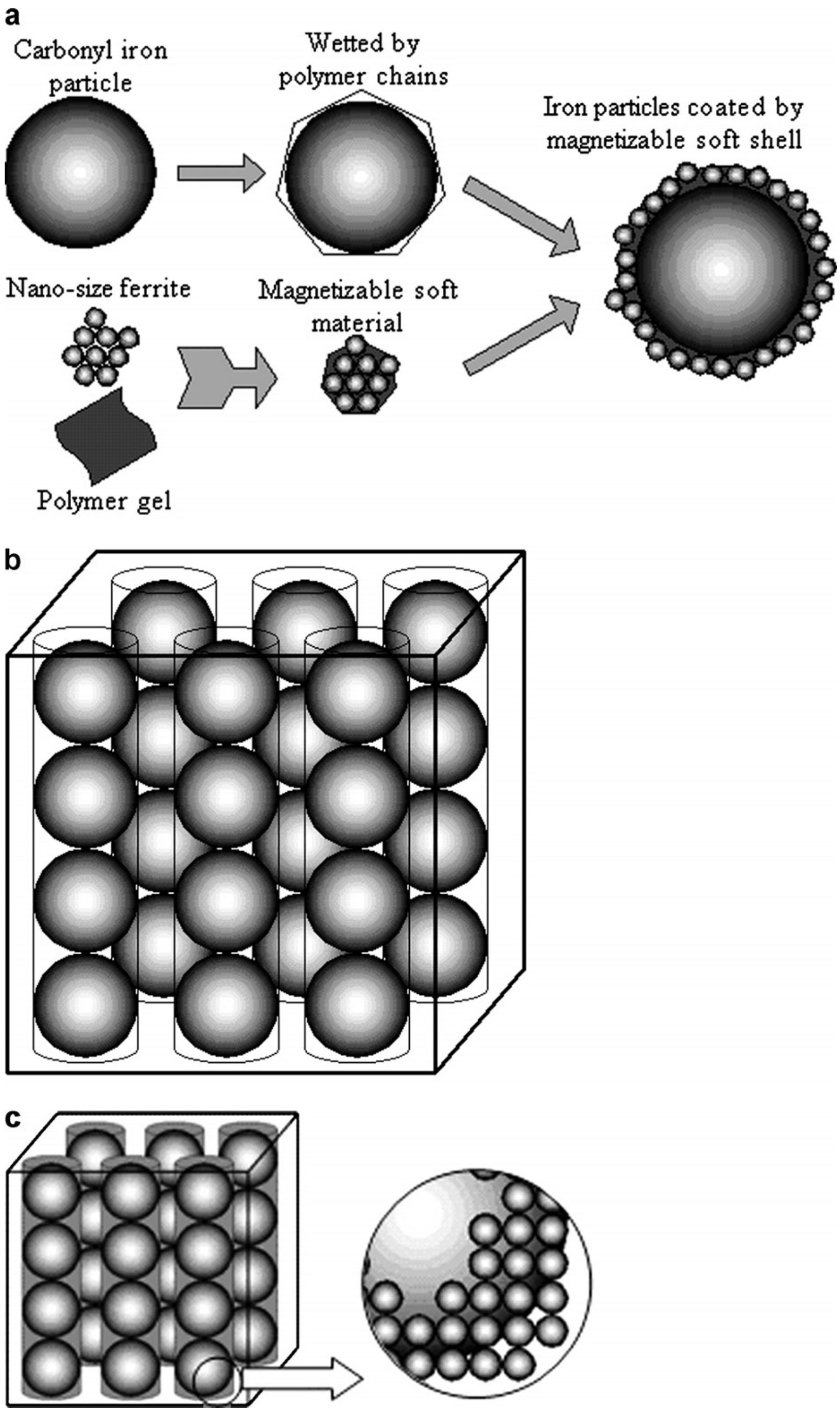

Fig. 1. New construction MREs: (a) Fabricate process, (b) traditional MR elastomers construction and (c) MR elastomers with nano-size particles additive.

Here, particles have permeability μ_p and that of matrix is μ_m, radius of particle is R, distance between adjacent particles is d, volume fraction of particles is ϕ_p, volume fraction of particles in column is $\phi = \frac{4R}{3d}$, volume fraction of column structure in MREs is $\phi_c = \frac{\phi_p}{\phi} = \frac{3\phi_p d}{4R}$. (The column is composed of the particles chain and the rubber between the particles.)

At the cross-section normal to the columns, the MREs are composed by columns domain and matrix domain. The effective permeability along the direction of columns is calculated by parallel-connection rule:

$$\mu_{\text{eff}} = \sum_{i=1}^{n} \phi_i \mu_i \quad \text{and} \quad \sum_{i=1}^{n} \phi_i = 1 \tag{2}$$

where the ϕ_i and μ_i are the volume fraction and relative permeability of component i.

So the effective relative permeability of conventional MREs is:

$$\mu_{\text{eff}} = \mu_{\text{ceff}}\phi_c + \mu_m(1-\phi_c) = \mu_m + 2\phi_p\mu_m \frac{\mu_p - \mu_m}{\mu_p + \mu_m - \frac{4R}{3d}(\mu_p - \mu_m)} \tag{3}$$

According to the equation: $\tau = -\frac{1}{2}\mu_0 \frac{\partial \mu_{\text{eff}}(\varepsilon)}{\partial \varepsilon} H_0^2$ [14] and $\varepsilon = \frac{x}{d}$ for shear mode (As shown in Fig. 2), the shear stress of MREs can be expressed as:

$$\tau = 12\phi_p\mu_0\mu_m\left(\frac{R}{d}\right)H_0^2 \frac{(\mu_p - \mu_m)^2\varepsilon}{\sqrt{1+\varepsilon^2}\left[3\sqrt{1+\varepsilon^2}(\mu_p + \mu_m) - 4\left(\frac{R}{d}\right)(\mu_p - \mu_m)\right]^2} \tag{4}$$

And shear modulus is:

$$G = 12\phi_p\mu_0\mu_m\left(\frac{R}{d}\right)H_0^2 \frac{(\mu_p - \mu_m)^2}{\sqrt{1+\varepsilon^2}\left[3\sqrt{1+\varepsilon^2}(\mu_p + \mu_m) - 4\left(\frac{R}{d}\right)(\mu_p - \mu_m)\right]^2} \tag{5}$$

Because $\varepsilon \ll 1$, $\mu_p \gg \mu_m$, and $R/d \approx 1/2$, the equation can be simplified as:

$$G \approx 6\phi_p\mu_0\mu_m H_0^2 \tag{6}$$

Conventional MREs is generally composed of magnetizable particles with an average diameter about several microns and polymer matrix such as rubber [2]. In this paper, a new material design is used to improve the performance of MREs. Different from conventional methods, the iron particles are coated with magnetizable soft shell composed of nano-size ferrite powder and polymer gel. As shown in Fig. 1a, in order to fabricate this kind of magentizble soft shell, nano-size ferrite and polymer gel are pre-requisite. Firstly, it is needed for forming a continuous composite structure to wet the particles by polymer chains. Then mix the nano-size particles

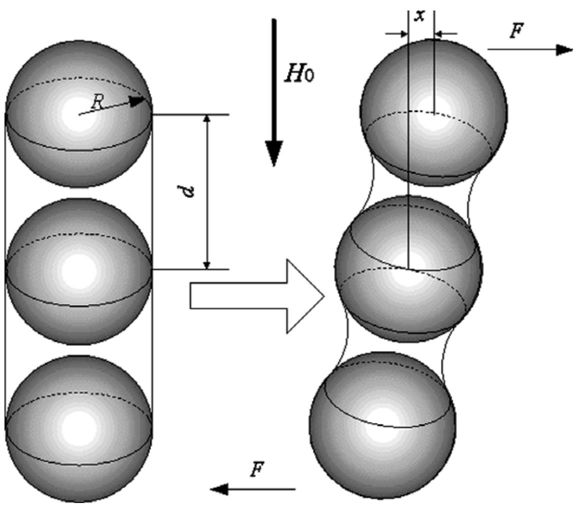

Fig. 2. The shear model of particles chain.

with polymer to produce the soft magnetism material. After that, coat the micro-size particles with the soft magnetism material and add this kind of coated particles into liquid rubber matrix to fabricate MREs. At last, the mixture is poured into a mould and a strong external magnetic field is applied to the mixture of particles and liquid rubber matrix to form chain like structure. With this process, magnetizable soft shell will deform and fill in the void space existing in micro-size particles chains. Thus, this method will increase the pack/energy density of MREs in special place and direction, and consequently will improve the shear modulus of MREs. The structural comparison between conventional MREs and the proposed new MREs is shown in Fig. 1b and c, where the nano-particles additives around micron-particles are zoomed out.

For novel MREs, the column structure without deformation is composed of micro-particles and soft shell. When MREs are deformed by force, the distance between particles is increased, some matrix around the column intrudes the column to refill the gap and the effective permeability is changed too. The changes result in the increase of magnetic energy and the field-dependent modulus of MREs (Fig. 2).

The magnetizable soft shell is composed of nano-particles and rubber and the volume fraction of nano-particles in soft shell is ϕ_n. At the shear mode, when the shear strain is $\varepsilon = \frac{x}{d}$, the volume fraction of particles (including nano-size and micro-size particles) in the column can be expressed as:

$$\phi_{pc} = \frac{4R + \phi_n(3d - 4R)}{3d\sqrt{1+\varepsilon^2}} \tag{7}$$

By using the same deduction as above-mentioned, the shear modulus of nano-additive MREs is:

$$G_n = \frac{3}{4}\phi_p\mu_0\mu_m\left(\frac{d}{R}\right)H_0^2 \frac{(\mu_p - \mu_m)^2\left[\frac{4R}{d}(\phi_n - 1) - 3\phi_n\right]^2}{\sqrt{1+\varepsilon^2}\left\{3\sqrt{1+\varepsilon^2}(\mu_p + \mu_m) + \left[\frac{4R}{d}(\phi_n - 1) - 3\phi_n\right](\mu_p - \mu_m)\right\}^2} \tag{8}$$

Because $\varepsilon \ll 1$, $\mu_p \gg \mu_m$, and $R/d \approx 1/2$, the equation can be simplified as:

$$G_n \approx \frac{3}{2}\phi_p\mu_0\mu_m H_0^2\left(\frac{2+\phi_n}{1-\phi_n}\right)^2 \tag{9}$$

When $\phi_n = 0$, Eq. (9) will degenerate to Eq. (6).

Actually, the relative permeability of particles is the function of magnetic field intensity and if the saturation ought to be taken into account, it is just need replace μ_p by $\mu_p(H)$ in Eq. (8), where $\mu_p(H)$ can be obtained by experiment.

Here an empirical equation about $\mu_p(H)$ is given as [15]

$$\mu_p(H) = \frac{H(\mu_p - 1) + \mu_p M_s}{H(\mu_p - 1) + M_s} \tag{10}$$

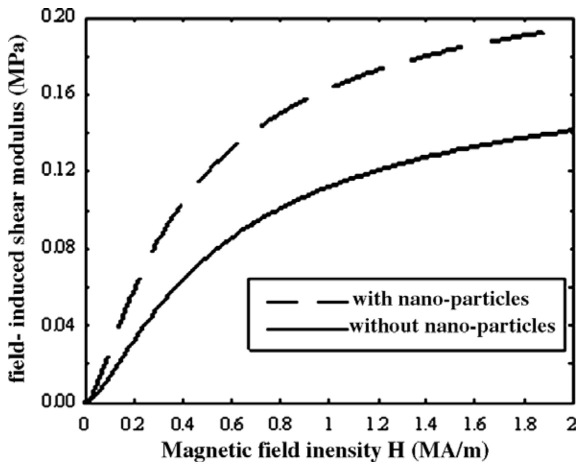

Fig. 3. The predicted field-dependent shear modulus.

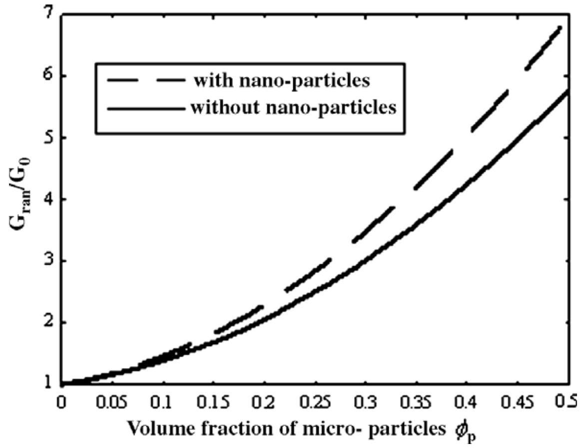

Fig. 4. The predicted zero-field shear modulus.

where μ_p is the largest relative permeability of particles, and M_s is the saturation magnetization and $\mu_0 M_s = 2.1$ T for Fe.

Assuming the relative permeability of particles is 1000 and that of matrix is 1, the volume fraction of micro-paritcles in MREs is 27% and the volume fraction of nano-particles in magnetizable soft shell is 27% too, the field-dependent shear modulus of conventional MREs and that of the novel MREs are calculated and shown in Fig. 3. Obviously, if the iron particles are covered by magnetizable soft shell, the shear modulus is increased significantly.

3. Zero-field shear modulus

The mechanical properties of MREs without a magnetic field can use the conventional model for prediction. The material can be seen as the composite properties with a different filler. The approximated shear modulus G_{ran} of elastomer filled with randomly distributed, spherical rigid particles is simply given by the equation [12]:

$$G_{ran} = G_0(1 + 2.5\phi_t + 14.1\phi_t^2) \tag{11}$$

where G_0 is the shear modulus of the unfilled elastomer and ϕ_t is the volume fraction of filler. The modulus calculated in this equation is similar to the value of anisotropic MREs [12].

From above analysis, the volume fraction of filler in novel MREs is $\phi_t = \frac{\phi_p[4R+\phi_n(3d-4R)]}{4R}$. Fixing the volume fraction of nano-size particles in film is 27% and $R/d = 1/2$, the zero-field shear modulus can be calculated and shown in Fig. 4. As can be seen from this figure, the added nano-size magnetism film can only minorly increase the zero-field shear modulus of MREs.

4. Conclusion

In this paper, a new effective permeability model, by taking into account the particle's saturation, is proposed to explain field-induced modulus of MREs with complex structure and components.

A novel structure MREs is introduced, as a case study, to verify the model. It is designed to improve the magnetic energy density and field-dependent performance. The new method uses the iron particles which are coated with magnetizable soft shell composed of nano-size ferrite powder and polymer gel. Mechanical performances of the newly proposed MREs are improved significantly.

The simulation results indicate that the novel MREs have the much larger field-dependent modulus than that of conventional ones. Meanwhile, the zero-field shear modulus of the MREs is not improved obviously by using the magnetizable soft shell.

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